In body-fixed axes a similar expression results. Since $\partial X/\partial u$ is the prime contributor, phugoid damping tends to zero not when $T_0 = D_0$, but when $\partial T/\partial u = \partial D/\partial u$.

Finally, the standard definition of a partial derivative holds constant all variables but one. Therefore it appears incorrect to include terms proportional to dr/du in the literal expressions of Table 1[1] for $\partial X/\partial u$ and $\partial Z/\partial u$.

All of these comments leave the general worth of Ref. 1 undiminished. They serve instead to sharpen its focus.

Reference

[1] Laitone, E. V. and Chou, Y. S., "Phugoid oscillations at hypersonic speeds," AIAA J. **3**, 732–735 (1965).

Reply by Authors to R. J. Woodcock

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The approximation to the phugoid period that is suggested by Woodcock[1] in his Eq. (2) gives the same numerical values as does our[2] Eq. (1.5) within 2% as long as $F < 0.99$, because for the earth's atmosphere ρ'/ρ is nearly 1000 times greater than $1/R$ at altitudes below 400,000 ft. Consequently the additional term in our[2] Eq. (1.5) is necessary only for the orbiting speed ($F = 1$) in order to give the correct earth satellite period. However, for other planetary bodies ρ'/ρ could become of the same order of magnitude as $1/R$, and then the additional term would become very important at hypersonic speeds.

Our[2] Sec. 2 was not based upon a constant engine thrust. As indicated by our Eq. (2.1) we derived Eqs. (1.5) and (2.3) by assuming that a varying thrust continually cancelled the total drag force, so that the condition derived by Woodcock,[1] following his Eq. (5), is automatically satisfied. Our Eq. (1.5) was derived in this manner so as to provide a direct comparison with the previous derivations (based on the same assumption of exactly zero net drag) of Lanchester [Eq. (1.1)] and Scheubel [Eq. (1.2)], and thereby explicitly delineate the new effects indicated by Etkin's[3] numerical calculations. We used Eq. (3.5) for the case of constant engine thrust, and in Sec. 3 we showed that whereas a finite drag force has only a slight effect upon the period, it directly controls all of the damping.

The expressions for X_u and Z_u in our[2] Table 1 must include the term dr/du if the flight trajectory is not parallel to the planet's surface because then r is a given function of u. For the cases considered in Etkin's[3] numerical calculations, the term dr/du can be neglected. However, this may not be the case for any steeply inclined trajectory of an aerodynamic body shape moving at hypersonic speeds through the earth's atmosphere. The term in question can be derived as follows:

$$X_u = -\frac{\partial}{\partial u}\left(\tfrac{1}{2}\rho V^2 S C_D\right)$$
$$= -\rho U S\left(C_D + \frac{U}{2}\frac{\partial C_D}{\partial u} + \frac{U C_D}{2\rho}\frac{\partial \rho}{\partial u}\right)$$
$$= -\rho U S C_D\left(1 + \frac{U}{2}\frac{1}{\rho}\frac{d\rho}{dr}\frac{dr}{du}\right)$$

since $\partial C_D/\partial u = 0$ for hypersonic speeds.

Our[2] Fig. 1 was calculated on the basis of the ρ'/ρ values used by Etkin[3] in his numerical calculations, whereas Woodcock's[1] Fig. 1 is based upon a constant value of $\rho'/\rho = -(22 \times 10^3)^{-1}$.

We would like to thank Mr. Woodcock for his comments because they have brought out some interesting details that we had not discussed. We also would like to note two misprints in our Eq. (1.4), which would have been printed as

$$T = (2)^{1/2}\left(\frac{U}{g}\right)\pi\left\{1 + \frac{U^2}{2g}\left(-\frac{\rho'}{\rho}\right) + \left[1 - \frac{U^2}{2g}\left(-\frac{a'}{a}\right)\right]\zeta\right\}^{-1/2}$$

References

[1] Woodcock, R. J., "Comments on 'Phugoid oscillations at hypersonic speeds,' " AIAA J. **4**, 762–763 (1966).
[2] Laitone, E. V. and Chou, Y. S., "Phugoid oscillations at hypersonic speeds," AIAA J. **3**, 732–735 (1965).
[3] Etkin, B., "Longitudinal dynamics of a lifting vehicle in orbital flight," J. Aerospace Sci. **28**, 779–788 (1961).

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Comment on "Prediction of Adiabatic Wall Temperatures in Film-Cooling Systems"

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Application of Spalding's combined boundary layer and jet-flow model correlation group for film cooling has been made to data from representative aircraft gas-turbine combustion chamber cooling devices, and comparison made between these data and Spalding's equation, which can be taken as applying for clean slot devices. It has been shown that over the appropriate range of values for such application, good correlation is not achieved, and the equation does not describe the data. A significant geometry effect is clearly shown to be present. An alternative correlation group is presented which, although not definitive, gives much improved results, and the resulting equation indicates that treatment of the film as more jet-like might be made for this type of data.

Spalding concludes that an artificial formula,[1] which is an uncomplicated combination of boundary layer and jet-flow model correlations, represents the best simple correlation formula to describe film cooling data over a wide range of velocity ratio. This correlation was tested against, and based on, simple geometry slots, and it was interesting to see how far it could be applied to dirty geometry slots,† which constitute most practical cooling systems. Accordingly, application of this formula has been made to the film cooling of aircraft gas-turbine combustion chambers. For such application, i.e., to a Mach 2.2 cruise supersonic transport typically, values of the group X are unlikely to exceed 10, and the film effectiveness usually must be limited to a minimum of about 0.6–0.7 because of considerations of permitted wall temperature.[2] X‡ was defined as

$$X = 0.91\left(\frac{u_g \cdot x}{u_c \cdot y_c}\right)^{0.8}\left(\frac{u_c \cdot y_c}{\nu}\right)^{-0.2} + 1.41\left(\left|1 - \frac{u_g}{u_c}\right|\frac{x}{y_c}\right)^{0.5} \quad ((1)$$

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† A dirty slot is defined as one with restricted inlet and/or outlet; it also may protrude into the mainstream. A clean slot is unrestricted and does not usually disturb the mainstream.

‡ Nomenclature as Spalding.

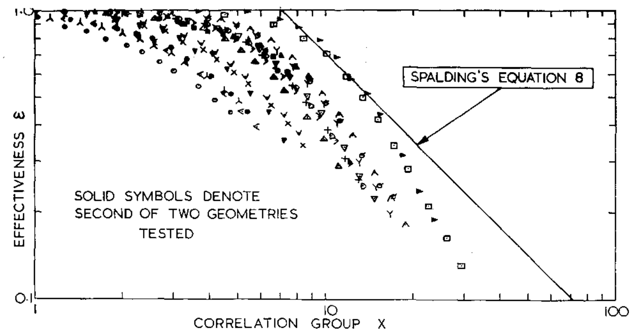

Fig. 1 Correlation of dirty slot data using X.

Data were obtained from two totally different dirty slots with geometries representing some of typical current European practice; by comparison, all the data used by Spalding can be considered to represent a clean slot. The range of velocity ratio covered was $0.195 \leqslant u_c/u_g \leqslant 2.35$ where velocities were based on mass flow rates and geometric areas; density and viscosity ratios, cold to hot, covered were from 1.8 to 2.7 and 0.54 to 0.65, respectively. Both mainstream and film were fully turbulent, the mainstream being heated upstream of the slot.

Figure 1 shows the degree of correlation achieved and compares the data with Spalding's Eq. (8)[1]; effectiveness in this case being based on wall temperatures. It can be seen that more complicated slot geometry decreases the length of potential core and initial mixing region by increasing the film turbulence, and results in a general displacement away from clean slot results; correlation of data cannot be considered satisfactory. The failure of correlation very close to the slot was to be expected; what is surprising is that this failure extends so far downstream. It can be attributed to the enlarged transition regions produced by the slot geometries and which are not covered by the equation. Note that for large values of X, the data indicate an improvement in correlation. These observations are in agreement with some data obtained for ramjet combustors with mainstream flows up to a Mach number of about 0.7. It might have been expected that the effect of dirty slot geometry would be solely a detrimental displacement from Spalding's Eq. (8), by a reduced potential core length for the film. However, it is clear that film development is more complex than this. It is unlikely therefore that adjustment of the constants in Spalding's formula might be made to account for the present data, as the lengths of potential core and transition region have been shown to be functions of geometry as well as velocity ratio. Currently favored geometries for chambers are such that fairly considerable variations in displacement of the eventual decay portion of the effectiveness curve can occur. Further, it has also been shown[2] that 70–100% of the distance cooled by a practical chamber film cooling device is within the potential core and extended disordered mixing region where similar profiles are being established, and not therefore really described by X.

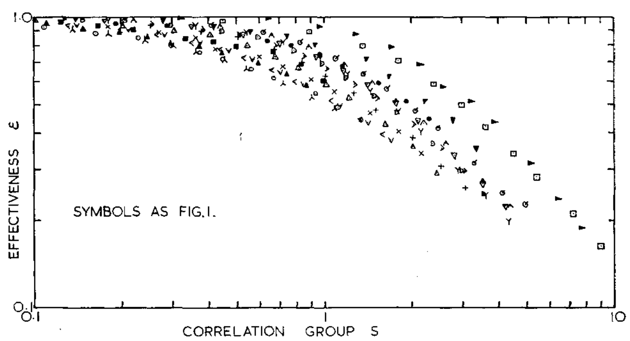

Fig. 2 Correlation of dirty slot data using S.

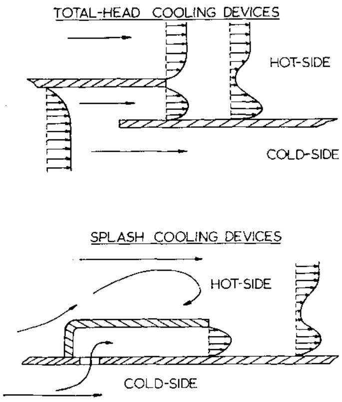

Fig. 3 Velocity profile development in practical film-cooling devices.

Over the region of interest for the film cooling of gas-turbine combustion chambers, better correlation of typical data, Fig. 2, has been obtained by the author using the semi-empirical group S, where

$$S = \{[(\rho_g u_g)/(\rho_c u_c)] \cdot (x/y_c)\} \cdot [(u_c y_c)/\nu]^{-0.25} \qquad (2)$$

This group is very similar to one obtained by Cole and Peerless[3] but lacks their viscosity ratio term; it does, however, give slightly better correlation. To give parity with Spalding's illustration, values of effectiveness were corrected to be unity at slot outlet since the initial coolant temperatures had to be measured just upstream. This results in a greater loss of correlation in S than in X, as no correction can be made to the correlating group. Spalding has made a similar effectiveness correction to the data of Papell et al.[4,5]

Linear plots confirm that the following equation describes the data down to an effectiveness of about 0.3;

$$\epsilon = C(S)^{-0.65} \qquad (3)$$

C being a constant apparently dependant on slot geometry primarily but also weakly on velocity ratio. The same slight improvement in correlation at small values of ϵ, i.e. large S, may be noted, implying a significant effect of geometry for some distance downstream of the slot outlet.

For the limited range of effectiveness of interest, it is really more meaningful to use the linear plots for data, but logarithmic plots have been used for direct comparison with X. In the linear form, correlation is more apparent for $1.0 \geqslant \epsilon > 0.3$; these plots have been omitted for the sake of brevity, but $S^{0.65}$ correlates effectiveness to ±4% just outside the potential core, and to ±16% at the limit of the data, for the geometries referred to.

Equation 3, applies well even to very small values of velocity ratio u_c/u_g, where the film might be expected to behave more like a boundary layer nearer to the slot outlet. The value of the exponent of S indicates that the film might be treated as more jet-like in the regions of potential core and initial mixing; values nearer in line with boundary layer flow were found only for $u_c/u_g <<< 1$ at $x/y_c > 20$, whereas, for $u_c/u_g \simeq 1$, nothing was detected up to $x/y_c = 60$. Values of effectiveness will usually be much less than the limiting value by x/y_c equal to about 40.

Equations (2) and (3) are by no means offered as definitive, but they serve the present purpose adequately. It is hoped that studies of the potential core and initial mixing region will, in the near future, yield further information on the "constant" C.

Practical cooling devices fall into two broad groups; those utilizing the full total pressure of the coolant flow, and those that are driven solely by the static pressure difference across the cooled walls. The two groups are commonly known as total head and splash devices, respectively. In Fig. 3, the probable velocity profiles existing in both types during normal operation for a velocity ratio $u_c/u_g < 1$ are sketched, showing how near the slot outlet at least, the film could be jet-like. Since it is necessary to add some kind of spacer in the slot to maintain correct gap heights during high temperature operation, the peakiness of the outlet velocity profile would tend to be further increased. If the flow pattern postulated for splash devices is correct, then the development of flow near the outlet from such slots should be less sensitive to velocity ratio than that from total-head devices. Measurements of potential core have shown this tendency, although, in fairness, this also could be because of other factors. The existence of initial velocity profiles in the film would result in extended transition regions, as indicated by Eq. 3.

For this particular application, it can thus be concluded that treatment of the cooling film as a turbulent boundary layer is unlikely to be rewarding, and development of different mass entrainment laws will be required for Spalding's new theory.[6] Investigations into these aspects are being carried out at this establishment as part of a broader program of combustion chamber cooling research. To paraphrase Spalding, obviously for dirty geometry slots, a flow that is of the boundary layer type away from the slot can be considered jet-like near the slot outlet, for the range of velocity ratios likely to be encountered. Application of Spalding's formula should be limited to the simple geometry slots for which it is valid and was intended, where initial velocity profiles in the film are vanishingly small and there is no wake from the lip of the slot.

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[2] Sturgess, G. J., "Application of film cooling theory to the cooling of aircraft gas turbine combustion chambers," Aeronautical Engineering Dept., Loughborough College, Loughborough, England (1965); unpublished work.

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Comments on "Stresses about a Circular Hole in a Cylindrical Shell"

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It seems to be worthwhile to add some remarks on the behavior of stresses around the hole for large values of the curvature parameter β.

The author states[1] that the Hankel functions of the first kind behave like

$$H_n^1 \sim Ce^{-\beta r}/(\beta r)^{1/2} \tag{33}$$

for large arguments, so that the stress function behaves like

$$\phi^* \sim Ce^{-\beta r(1 - |\cos\theta|)}/(\beta r)^{1/2} \tag{34}$$

This implies that the stresses caused by the hole at the 90° position decay most rapidly.

On the other hand, the boundary-layer analysis results in the following behavior of the stress function ϕ^* for large β:

$$\phi^* \sim Ce^{-2(1-i)\beta r \cos\theta} \qquad \text{for} \qquad \cos\theta > 0 \tag{52}$$

thus implying that stresses caused by the hole at the 0° position decay most rapidly, whereas, on the contrary at $\theta = 90°$, a turning-point-type singularity appears. This conclusion seems to be the correct one. It was to be expected on the basis of the general asymptotic theory of shells and the singularity was pointed out, for instance, in Ref. 2.

The contradiction between (34) and (52) arises because the asymptotic formula (33) is valid for large values of the argument provided the order n of the Hankel function is small.†[3]

However, since "as β was increased, the number of terms needed in the series to obtain converging results also increased,"[1] and, for $\beta = 4$ for instance $n \leqq 28$, the asymptotic formula (33) remains valid only at very large distances from the hole, where $r \gg 1$, and not near the hole, where $\beta r \approx \beta$.

References

[1] VanDyke, P., "Stresses about a circular hole in a cylindrical shell," AIAA J. **3**, 1733–1742 (1965).

[2] Goldenveizer, A. L., *Theory of Elastic Thin Shells* (Pergamon Press, New York, 1961), p. 476.

[3] Watson, G. N., *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, New York, 1958), Chap. VIII.

† This was pointed out to the author of this note by D. A. Ludwig of the Courant Institute.

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Reply by Author to A. Kornecki

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The author is indebted to A. Kornecki for mentioning a point that was omitted in the original paper: the behavior of the stresses are characterized by the behavior of Eq. (34)[1] in the region near the hole when the parameter β is small enough so that the order n of the last term in the required Hankel function series also is small. This behavior, where the stresses at the 90° position decay most rapidly, is still evident at a β of 2(an n of 9) as indicated by Figs. 6 and 11; the stresses have reached the values of the membrane solution most quickly at the 90° position as compared with the 45° and 0° positions.

Further illumination regarding the behavior of the stresses in the shell as β becomes large is clearly necessary. The stress behavior in possible side ($|\cos\theta| > 0$) boundary layers, as characterized by Eq. (52), is, however, of interest only in the case of pressure loading; this side boundary-layer behavior is absent in the cases of loading by tension and torsion. A complete evaluation of the decay of the stresses at the hole edge, with the resulting determination of the point at which this decay is most rapid, would still appear to the author to depend upon a solution to the differential equation which governs the boundary layer at the 90° positions on the hole

$$\Phi^*_{,\bar y\bar y\bar y\bar y} + 8i(\Phi^*_{,\bar x\bar x} + \bar x^2\Phi^*_{,\bar y\bar y} + 2\bar x\Phi^*_{,\bar x\bar y} + \Phi^*_{,\bar y}) = 0 \tag{58}$$

under the boundary conditions [Eqs. (59) and (67)] appropriate to the loading.

Reference

[1] VanDyke, P., "Stresses about a circular hole in a cylindrical shell," AIAA J. **3**, 1733–1742 (1965).

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